Journal of Applied Mechanics and Technical Physics, Vol. 51, No. 1, pp. 84–88, 2010

THEORETICAL MODELS OF ACOUSTIC EMISSION IN ROCKS WITH DIFFERENT HEATING REGIMES

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UDC 622 : 550.372

Mechanisms of the emergence of mechanical stresses in rocks with different regimes of their heating are studied. Theoretical models of acoustic emission induced by these stresses are justified and analyzed.

Key words: *theoretical model, thermoacoustic emission, thermal stresses, temperature field.*

Introduction. The reason for the emergence of acoustic emission (AE) in rocks is the formation of new cracks or the growth of already existing cracks under the action of mechanical stresses. If such stresses are induced by temperature effects, they are usually called the thermal stresses and the corresponding emission is called the thermoacoustic emission.

One possible mechanism of the emergence of thermal stresses induced by a temperature gradient on the edges of cracks separating structural elements of the geomaterial was considered in [1]. That paper also justified the corresponding theoretical model of thermoacoustic emission (TAE) and the thermoemission memory effect arising in the course of cyclic heating of rocks with the maximum temperature being increased from one cycle to another [2]. That model, however, was not perfect because it could not give a clear explanation for the experimentally observed fact of higher values of TAE parameters in polymineral media than in monomineral media; moreover, the model did not explain the influence of the rate of the temperature increase in the sample on TAE parameters [2, 3].

The present paper offers justification of TAE theoretical models whose specific feature is the allowance for thermal stresses induced by the difference in the thermal coefficients of volume expansion (TCVE) of individual mineral grains forming the rock and by the nonuniformity of the temperature field in the sample.

1. Thermoacoustic Emission in a Uniform Temperature Field in the Examined Sample. If there are no temperature gradients in a geomaterial consisting of elements with different TCVEs, the only parameter determining TAE is the current temperature. The greater the difference between the current temperature and the initial temperature (at which the absence of mechanical stresses inside the mineral grains and on their boundaries is assumed), the greater the values of local stresses and the higher the probability of the growth of existing cracks and the formation of new cracks; as a consequence, the higher the degree of the acoustic emission. As the value of thermal stresses is proportional to the temperature difference, the derivatives of the total acoustic emission N_Σ with respect to temperature can be assumed to have close values for different rates of temperature variation.

If the rocks are considered as ideally elastic and brittle media and the redistribution of stresses due to the formation of microscopic defects is neglected, then the growth of the existing cracks and the formation of new cracks (accompanied by acoustic emission) during cyclic heating of the rocks occur only on exceeding the maximum temperature reached during the entire process, which determines the mechanism of the thermoemission memory effect.

Model No. 1. To obtain a qualitative estimate of TAE, we consider a simple model where an individual grain is an inclusion in a homogeneous matrix whose TCVE differs from that of the grain. During an increase

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in temperature, the ambient medium is assumed to affect the grain owing to thermal expansion of the matrix substance. Then, the inclusion is subjected to a stress (compression) equal to

$$\sigma = \Delta\alpha\, E\, \Delta T. \tag{1}$$

Here, $\Delta\alpha$ is the difference in TCVEs of the ambient substance (matrix) and inclusion, E is the modulus of elasticity of the inclusion, and ΔT is the difference between the current temperature and the temperature at the initial state.

In the case considered, the boundary of the external region experiences a stress from the side of the inclusion. There are also shear stresses, which decrease with distance from the inclusion. At large distances ($r > 3r_0$), all components of stresses, which arise owing to the presence of such a stress concentrator, have an asymptotic solution of the form

$$\sigma_e \sim \frac{\sigma}{(r/r_0)^3}\, f(\theta, \varphi), \tag{2}$$

where r is the distance from the inclusion, r_0 is the characteristic size of the inclusion, and $f(\theta, \varphi)$ is a function depending on the location of the point considered with respect to the inclusion (θ and φ are the longitude and latitude in a spherical coordinate system, respectively).

Let us assume that the growth of the existing cracks and the formation of new cracks and, hence, AE in the rocks occur when the stresses reach a certain critical value $\sigma_{\rm cr}$. Because of the temperature effect, a region with the stresses higher than $\sigma_{\rm cr}$ is formed around each inclusion; the volume of this region increases with increasing ΔT. Using Eqs. (1) and (2), we can estimate the size of this region:

$$r \sim \Big(\frac{\Delta\alpha\, E\, \Delta T\, r_0^3}{\sigma_{\rm cr}}\, f(\theta, \varphi)\Big)^{1/3}.$$

The expression for the volume occupied by this region bounded by the surface S is written as

$$V \sim \Big(\frac{E r_0^3}{\sigma_{\rm cr}} \int\limits_S f(\theta, \varphi)\, ds\Big) \Delta\alpha\, \Delta T. \tag{3}$$

Assuming that the total acoustic emission N_Σ is proportional to the volume of the region where the stresses exceed $\sigma_{\rm cr}$, we obtain

$$N_\Sigma(\Delta T) = \xi\, \Delta T, \tag{4}$$

where ξ is the proportionality coefficient depending on the expression in the brackets in Eq. (3). The coefficient ξ should be refined with allowance for the region geometry and elastic properties. As the considerations discussed here have a qualitative character, however, the value of ξ should be determined experimentally.

Model No. 2. The model considered above can be refined if we assume that the stresses inside the inclusion are obtained by solving the problem with elastic rather than stiff boundary conditions. Let us assume that the contact stresses on the grain boundary σ_{ij} equal to the stresses inside the inclusion are determined from the Eshelby problem of stresses in an inclusion made of a material whose properties differ from the properties of the matrix material and which has its own strains ε_{ij}^0 (caused by temperature stresses in the case considered):

$$\sigma_{ij} = E_{ijkl}^0 (S_{klpq} \varepsilon_{pq}^* - \varepsilon_{kl}^0). \tag{5}$$

In Eq. (5), the components of the tensor ε_{mn}^* are determined by solving the system [4, 5]

$$E_{ijkl}^1 (S_{klmn} \varepsilon_{mn}^* - \varepsilon_{kl}^0) = E_{ijkl}^0 (S_{klmn} \varepsilon_{mn}^* - \varepsilon_{kl}^*),$$

where E_{ijkl}^0 and E_{ijkl}^1 are the tensors of the moduli of elasticity of the matrix and inclusion, respectively, and S_{ijkl} are the components of the Eshelby tensor, which relate the confined strain in the inclusion ε_{kl}^* to the free strain ε_{kl}^0 in it [4]. In this case, the inherent strains are determined only by the difference of the TCVEs of the matrix and inclusion $\Delta\alpha_{ij}$, which is a tensor of the second rank in the general case:

$$\varepsilon_{ij}^0 = \Delta\alpha_{ij}\, \Delta T. \tag{6}$$

It should be noted that Eqs. (5) with allowance for Eqs. (1) and (6) differ only by the coefficient, which should be determined experimentally by virtue of the qualitative character of the models considered. Therefore, the expression for the total acoustic emission N_Σ (4) is also valid for model No. 2 with accuracy to a constant factor.

Model No. 3. One type of stress concentrators is the corner points of the boundary separating materials with different elastic properties. Under external loading, the distributions of stresses have a power-type singularity near the ribs of dihedral corners:

$$\sigma_e \sim \frac{\Delta\alpha\,\Delta T}{r^k}\, f(\varphi).$$

Here, $f(\varphi)$ is a function depending on the position of the point considered inside the dihedral corner of the interface. Assuming, as in model No. 1, that the growth of the existing cracks and the formation of new cracks, as well as acoustic emission occur in a certain region where the stresses exceed $\sigma_{\rm cr}$, we can estimate the linear size of this region as

$$r \sim \Big(\frac{\Delta\alpha\,\Delta T}{\sigma_{\rm cr}}\, f(\varphi)\Big)^{1/k}.$$

As this region is a dihedral corner, its volume can be estimated as

$$V \sim \Big(\frac{l}{\sigma_{\rm cr}^{2/k}} \int\limits_S f^{2/k}(\varphi)\, ds\Big)\Delta\alpha^{2/k}\, \Delta T^{2/k},$$

where l is a parameter that has the dimension of length. As in model Nos. 1 and 2, the total AE is a function of the temperature difference:

$$N_\Sigma(\Delta T) = \xi'\, \Delta T^\gamma$$

(the factor $\gamma = 2/k > 4$ and the proportionality coefficient ξ' are determined from experiments).

Thus, the stresses determined for each model are proportional to the product of the difference in the temperature coefficients of volume expansion of the inclusion and the matrix and the difference between the initial and current temperatures. The volume of the region where the stresses are higher than the critical value and acoustic emission can arise is determined by a power function of the temperature difference. The index of the power function is $\gamma = 1$ for model Nos. 1 and 2 and $\gamma > 4$ for model No. 3.

Note that the value $\gamma = 1$ corresponds to the asymptotic solution for the far (from the inclusion) field, while the value $\gamma = 4$ corresponds to the asymptotic solution for the near field; in reality, therefore, we can expect that $1 < \gamma < 4$.

It should also be noted that there may exist several groups of inhomogeneities with different TCVEs in the rock. In addition, the distribution of stresses around the grain is affected by the grain shape. Finally, local strength properties of the rock have a certain statistical distribution. Therefore, we can expect a joint action of these factors and, as a consequence, strong nonlinearity of the dependence of acoustic emission on the temperature difference.

2. Thermoacoustic Emission in a Nonuniform Temperature Field in the Examined Sample. In the case of nonuniform heating, where the temperature effect depends on the direction in space, there appears an additional source of stress concentration owing to the temperature field gradient. At the unsteady stage of heating, acoustic emission can be expected to increase, with its subsequent reduction as the temperature becomes equalized over the volume.

To calculate the stresses in the case considered, we have first to solve an unsteady problem of heat conduction and then, using the temperature field found at the first stage, calculate the stresses by equations of thermal elasticity. The stresses in a cylindrical sample of radius R are determined by the formulas [6]

$$\begin{aligned} \sigma_r(\rho) &= \frac{\alpha E}{\rho^2}\Big(\rho^2 \int\limits_0^1 T(\rho)\rho\, d\rho - \int\limits_0^\rho T(\rho)\rho\, d\rho\Big), \\ \sigma_\theta(\rho) &= \frac{\alpha E}{\rho^2}\Big(\rho^2 \int\limits_0^1 T(\rho)\rho\, d\rho + \int\limits_0^\rho T(\rho)\rho\, d\rho - \rho^2\, T(\rho)\Big). \end{aligned} \tag{7}$$

Here, $\sigma_r(\rho)$ and $\sigma_\theta(\rho)$ are the radial and circumferential components of the normal stresses, respectively, and $\rho = r/R \leqslant 1$ is the dimensionless coordinate.

If the temperature of the cylindrical surface of the sample changes with time t by the law $T(R,t) = kt$, where k is the rate of sample heating, the distribution of the temperature increment with the end-face effects being ignored can be determined, following [7], by the formula

$$T(r) = k\Big(t - \frac{R^2 - r^2}{4a}\Big) + \frac{2k}{Ra} \sum_{n=0}^{\infty} \mathrm{e}^{-a\alpha_n^2 t} \, \frac{J_0(r\alpha_n)}{\alpha_n^3 J_1(R\alpha_n)}. \tag{8}$$

Here, a is the thermal diffusivity, $J_0(r\,\alpha_n)$ and $J_1(R\alpha_n)$ are the Bessel functions of the zeroth and first kind, respectively, and α_n are the positive roots of the equation $J_0(a\alpha_n) = 0$.

At high times corresponding to the steady heating process, all terms of the series containing exponents in Eq. (8) can be neglected. The temperature and stress distributions corresponding to the steady state can be found by substituting the relation $T(\rho) = k[t - (R^2 - \rho^2)/(4a)]$ into Eq. (7):

$$\sigma_r(\rho) = \frac{k\alpha E R^2}{16a}\,(1 - \rho^2), \qquad \sigma_\theta(\rho) = \frac{k\alpha E R^2}{16a}\,(1 - 3\rho^2). \tag{9}$$

It follows from Eqs. (9) that the critical stress state is determined by the thermal diffusivity, sample size, and heating rate and does not depend on the conditions of heat exchange on the boundary.

Based on the estimates obtained, we can conclude that the level of mechanical stresses and the associated AE are not uniquely determined by the current temperature even if its variations in the experiment are specified by simple programs.

Thermoacoustic memory effects in the steady and transitional regimes can be less expressed because of the influence of different mechanisms of their emergence.

3. Comparison of the Effect of Various Mechanisms on the TAE Level. It is incorrect to compare the stresses caused by the global nonuniformity of the temperature field with the stresses arising on the grain boundaries because of the difference in TCVEs in a uniform temperature field. It seems reasonable to compare the stresses (1) with the stresses on the concentrators arising under the action of the stresses (9). In this case, we can estimate the stresses induced by the temperature gradient:

$$\sigma_g \sim \alpha\,\Delta E\,\frac{kR^2}{16a}. \tag{10}$$

A comparison of Eqs. (1) and (10) shows that prevailing of a particular mechanism is determined by the heating rate. Let us write the relation

$$\frac{\sigma_g}{\sigma} = \xi''\,\frac{\alpha}{\Delta\alpha}\,\frac{\Delta E}{E}\,\frac{kR^2}{16a\,\Delta T}, \tag{11}$$

where ξ'' is the proportionality coefficient close to unity in the order of magnitude.

If Eq. (11) is greater than unity, then the process caused by the global nonuniformity of the temperature field prevails. Let us estimate relation (11). In real rocks, the moduli of elasticity may change in a much greater interval than the TCVE, i.e., $(\alpha/\Delta\alpha)(\Delta E/E) > 1$. The quantity $kR^2/(16a)$ is the difference in temperatures at the sample center and on its surface; therefore, we have $kR^2/(16a\,\Delta T) < 1$ in all cases. Thus, realization of the TAE mechanism that is not caused by the global nonuniformity of the temperature field requires rather low rates of sample heating.

4. Conclusions. The theoretical models justified above show that different mechanisms of the formation of thermal stresses can be developed in rocks under the action of temperature effects; the emergence of thermoacoustic emission is caused by the joint action of these mechanisms. At the same time, the contributions of all mechanisms to the total AE are different and are determined both by thermophysical properties of structural elements of the geomaterial and, to a large extent, by the heating rate. Therefore, the analysis of AE dynamics with variations of the heating rate allow one to reveal the prevailing mechanism of the emergence of thermal stresses and to estimate the degree of uniformity of thermophysical properties of polymineral aggregate elements. For a particular rock, obviously, there should exist optimal heating rates that ensure a smaller or greater degree of manifestation of some particular thermoemission effects (in particular, thermoemission memory effect). The rate of the temperature increase should be taken into account in identifying the type of rocks and their belonging to a particular deposit on the basis of thermoemission passports, i.e., the heating rates should be identical in tests of the same type.

This work was supported by the Russian Foundation for Basic Research (Grant No. 07-05-00045).

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